Australian National University

Time reverse imaging of tsunami waveforms

Jakir Hossen[1,2], Phil R Cummins[2] and Jan Dettmer[2]
[1]BRAC University, Dhaka, Bangladesh
[2]Research School of Earth Sciences, Australian National University
Canberra ACT Australia

(Collaborators: Toshitaka Baba, Sebastian Allgeyer)

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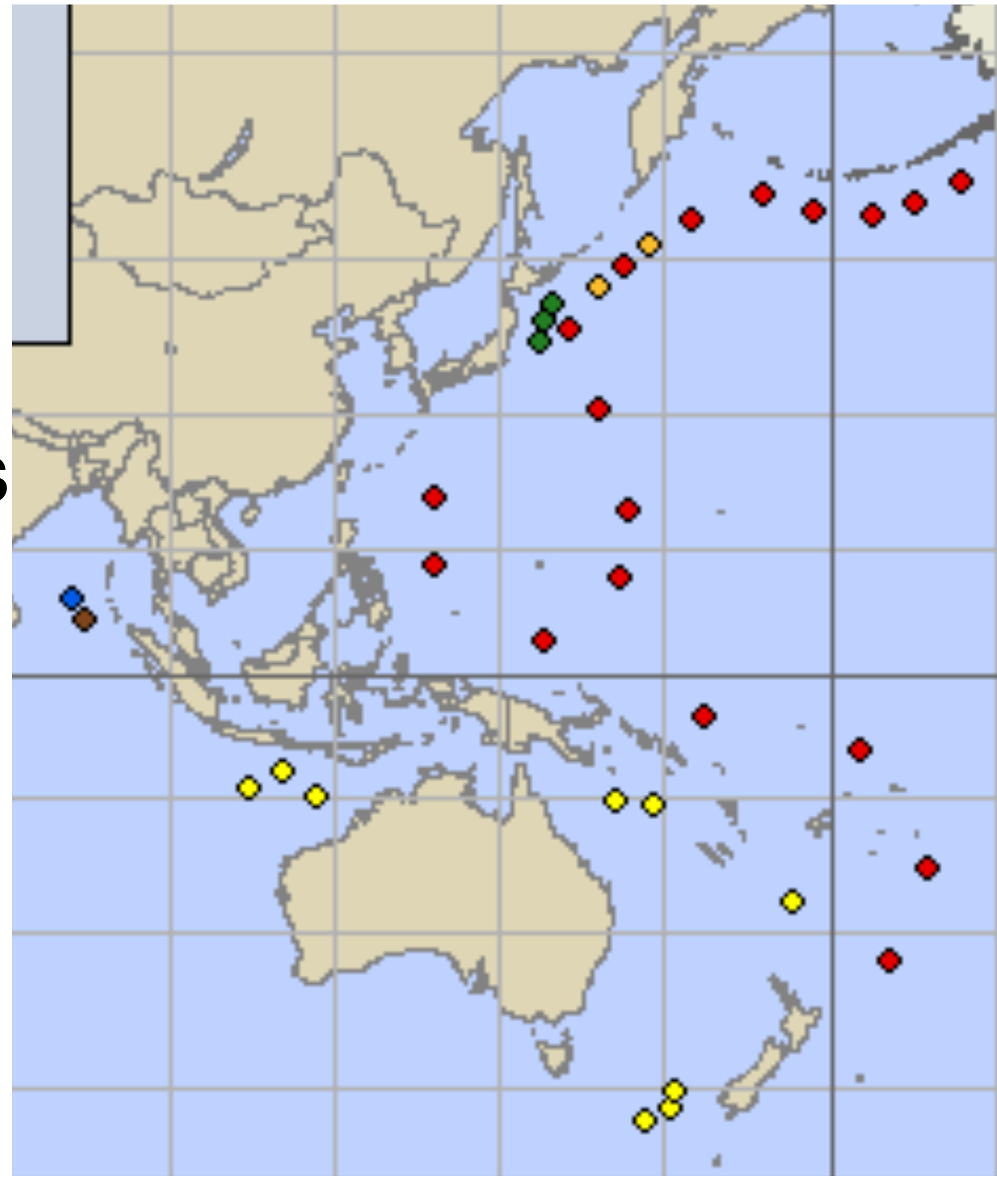
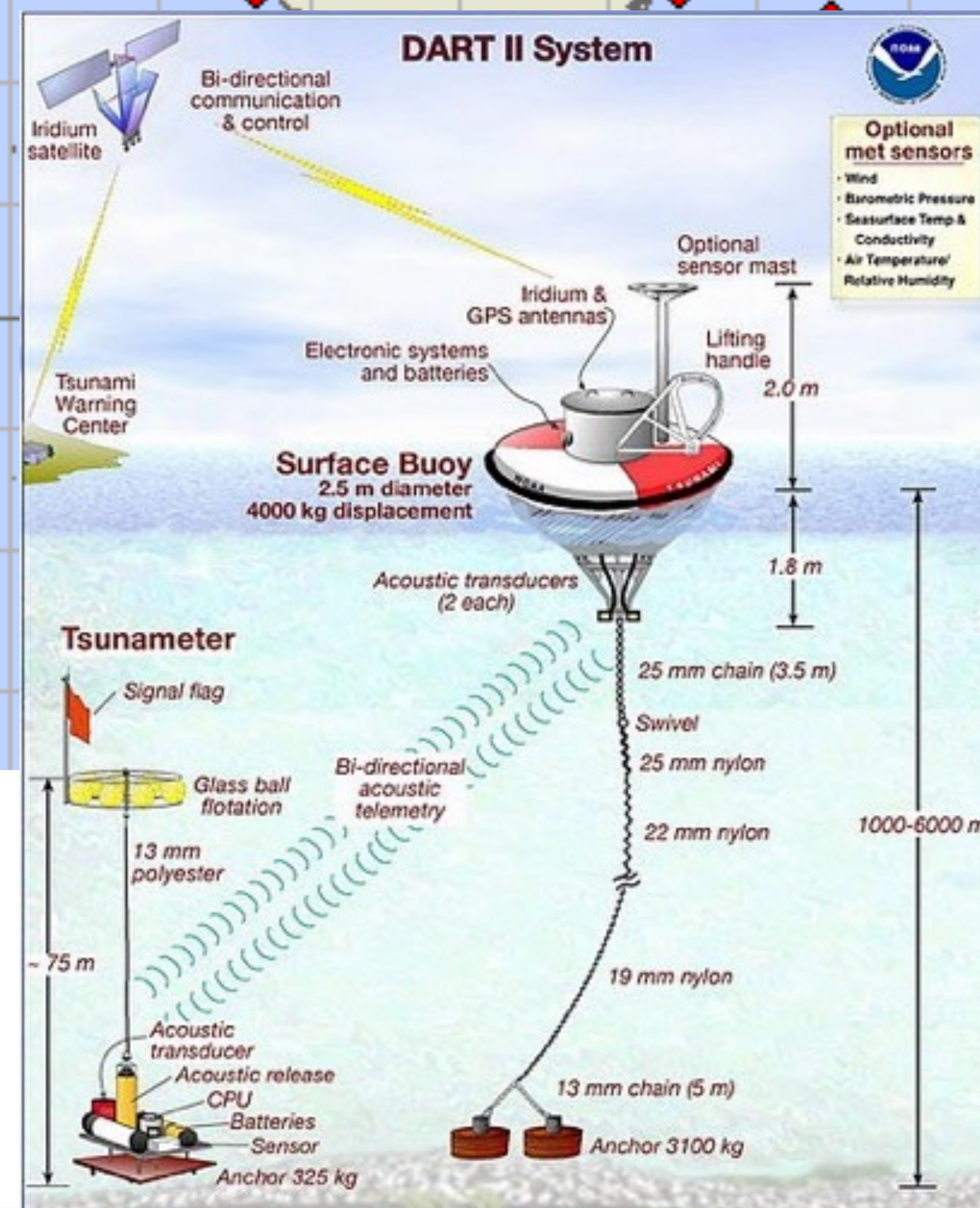
New Frontiers in Tsunami Observation: Global DART Buoy Network & Friends

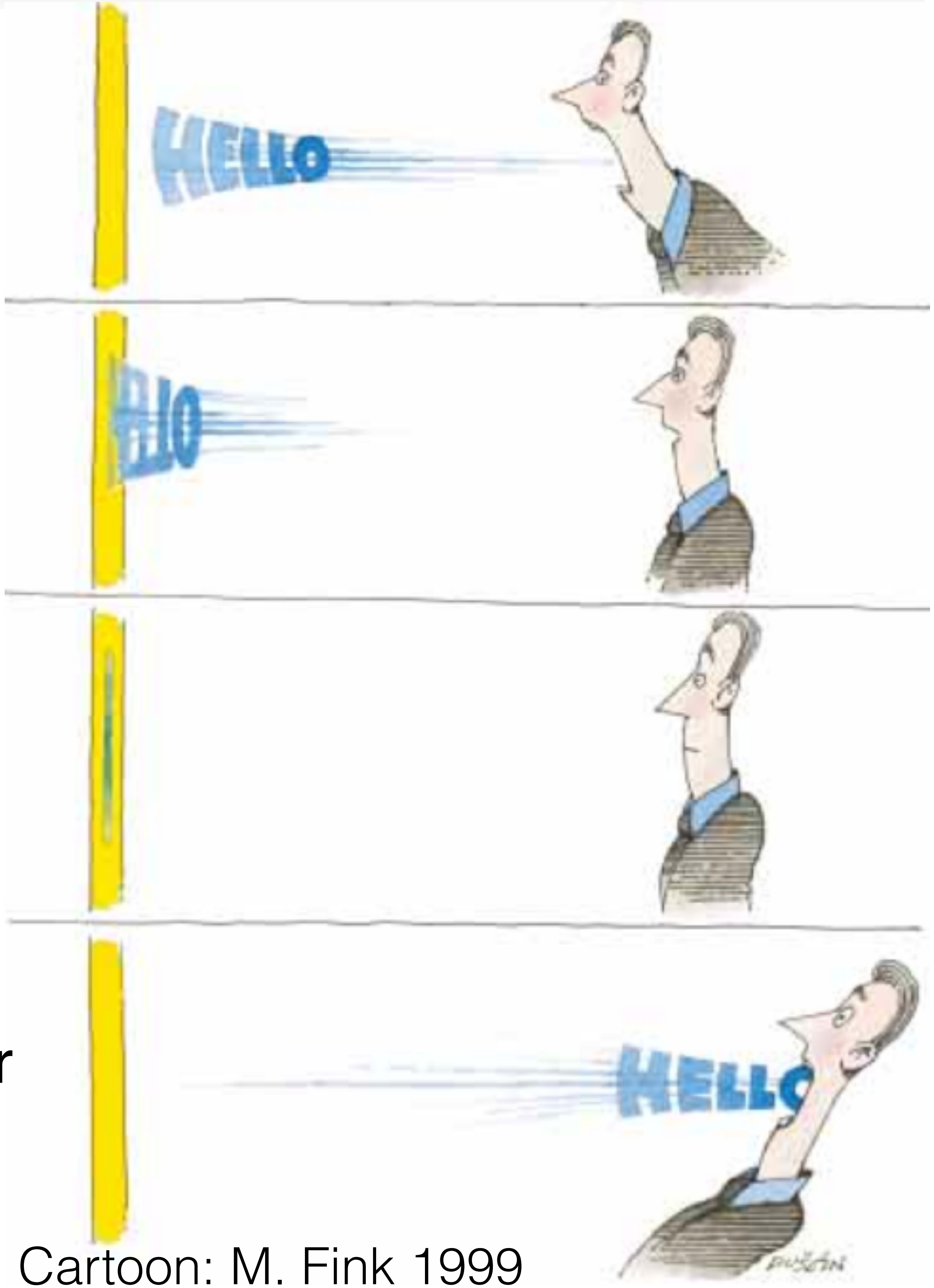




- Like hard-rock seismometers, deep-ocean tsunami measurements suffer only benign path effect
- They give much clearer picture of the tsunami source than coastal tide gauges.
- The new global network of buoys should revolutionize tsunami source Imaging.

Time reverse imaging (TRI) can reproduce source

- Common in acoustics, medical science, material science… (long distance communication, destruction of tumours & kidney stones, detection of material defects)
- Basis: **The physical processes underlying wave propagation are unchanged if time is reversed**
- Time-reversed signal: Refocuses @ location of original source regardless of the complexity of the propagation medium, with a similar shape (depending number of sensors)



Cartoon: M. Fink 1999

Tsunami propagation: Shallow water wave equations

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0$$

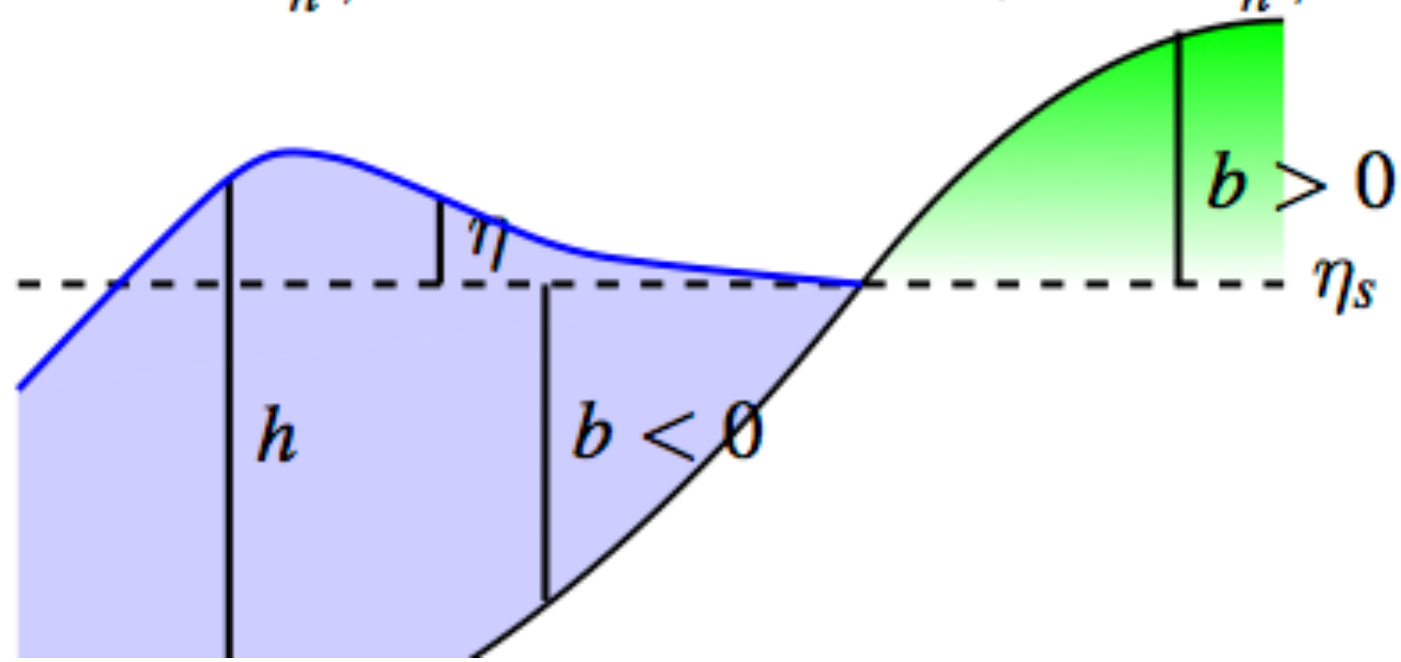
$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left(u^2 h + \frac{1}{2} g h^2\right) + \frac{\partial (huv)}{\partial y} = -gh\frac{\partial b}{\partial x} - ghD_x$$

$$\frac{\partial (hv)}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (v^2 h + \frac{1}{2} g h^2)}{\partial y} = -gh\frac{\partial b}{\partial y} - ghD_y$$

where $b(x, y)$ is the bed elevation and D is the bed friction.

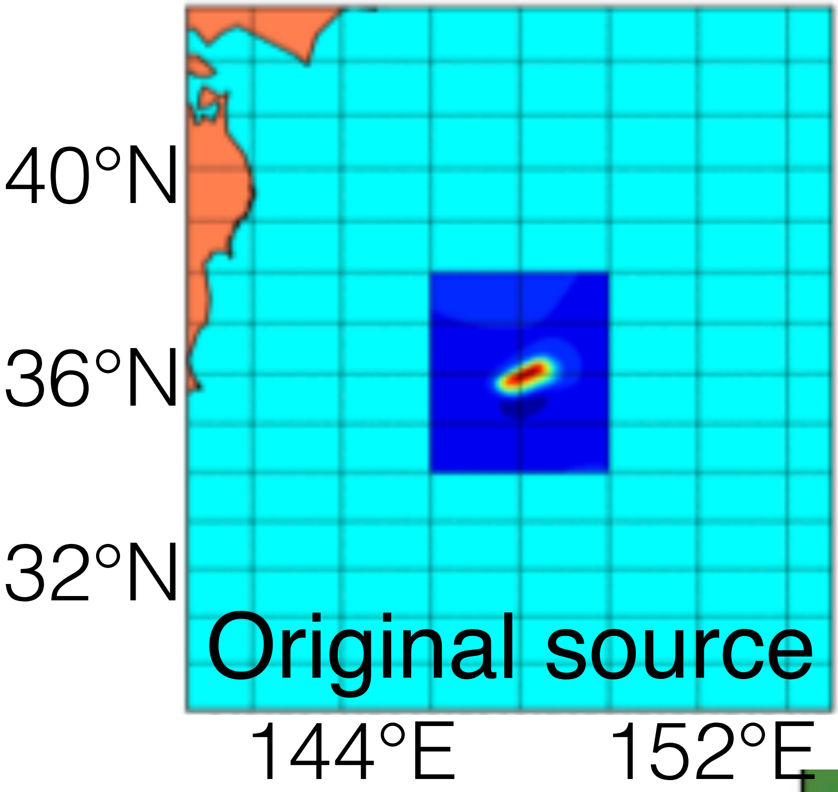
$$D_x = \frac{u\zeta^2\sqrt{u^2+v^2}}{h^{8/3}} \quad \text{and} \quad D_y = \frac{v\zeta^2\sqrt{u^2+v^2}}{h^{8/3}}$$

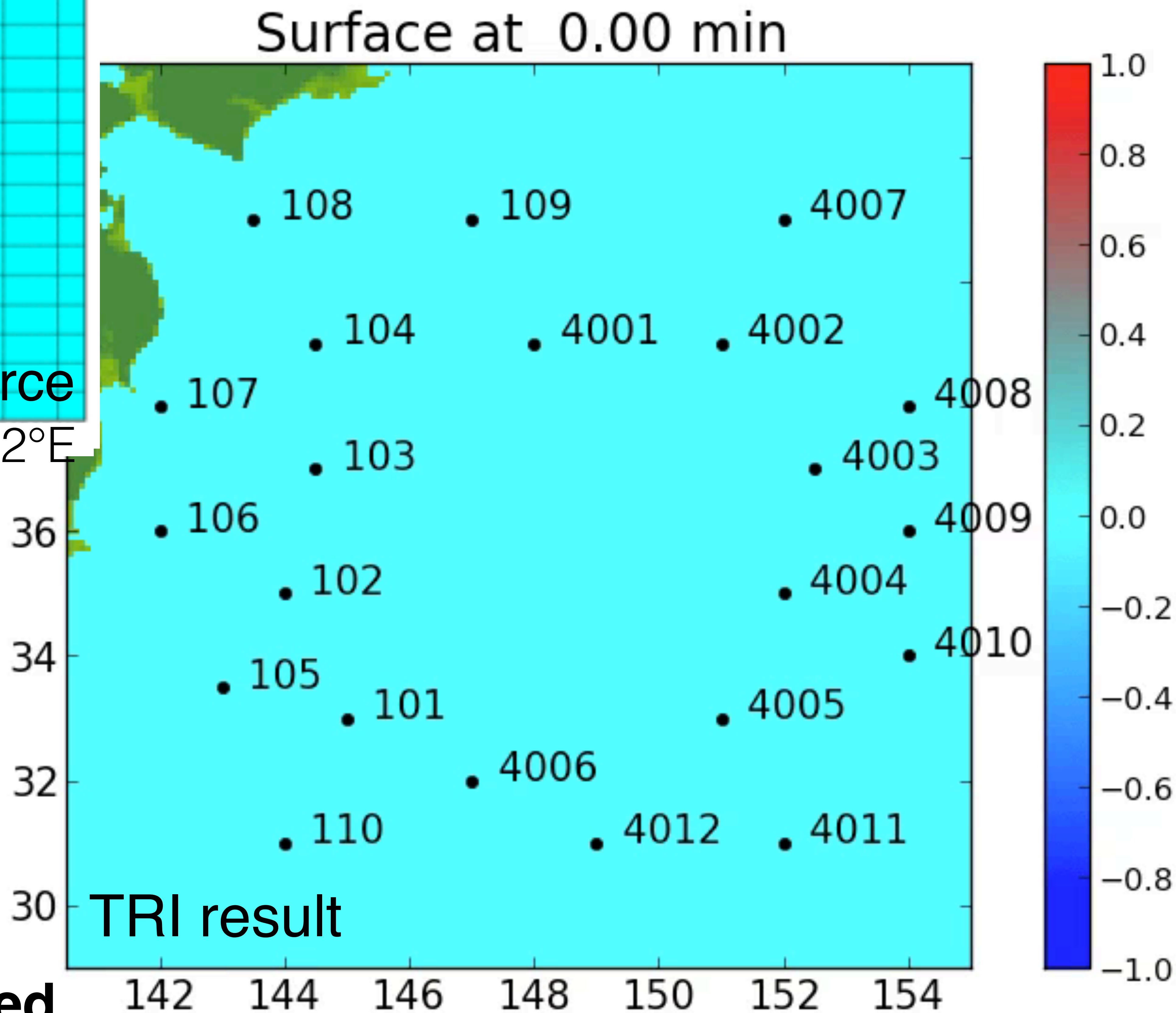
- All terms involving t, u, v change sign under time reversal
- Equations are invariant if the **bed friction** terms (D) are dropped



Details: Hossen et al. Pure & Applied Geophysics (2015)

Tsunami TRI: Simulation with encouraging results

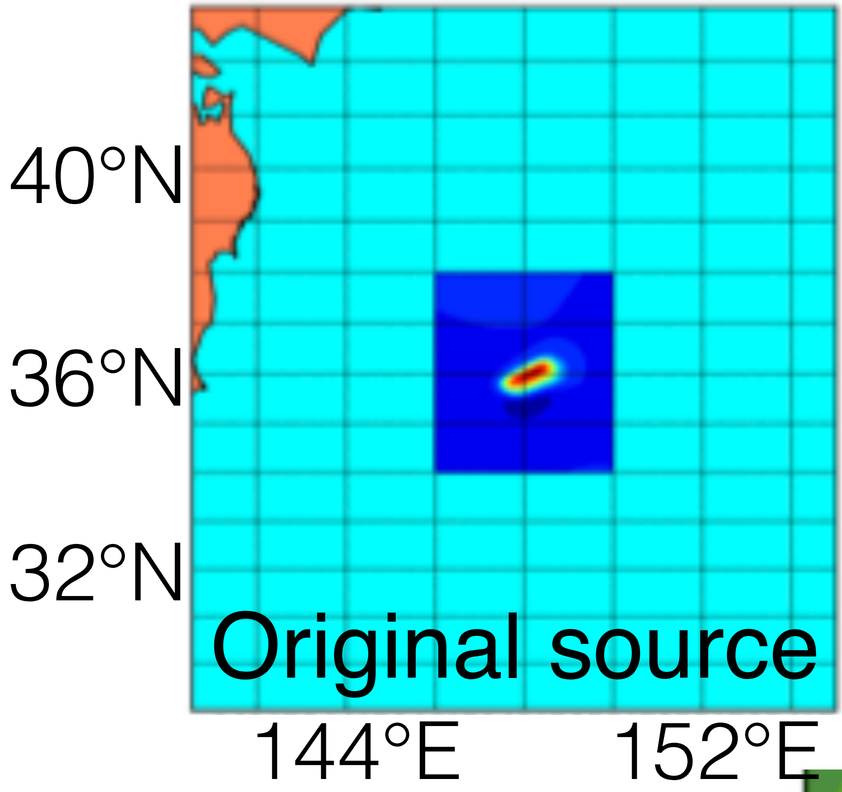


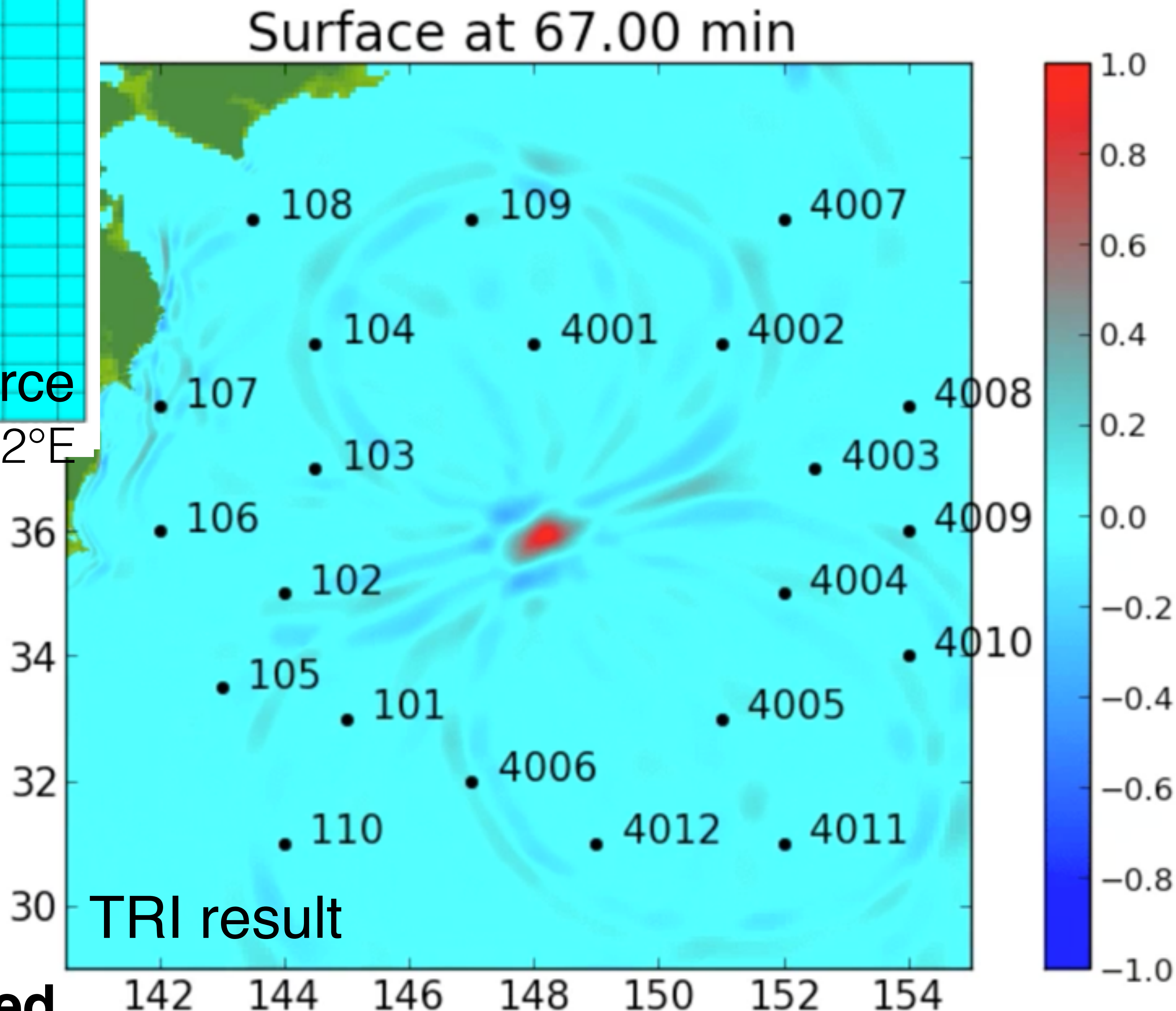


Simple, simulated illustration works…

Problem: Real tsunamis much more complicated

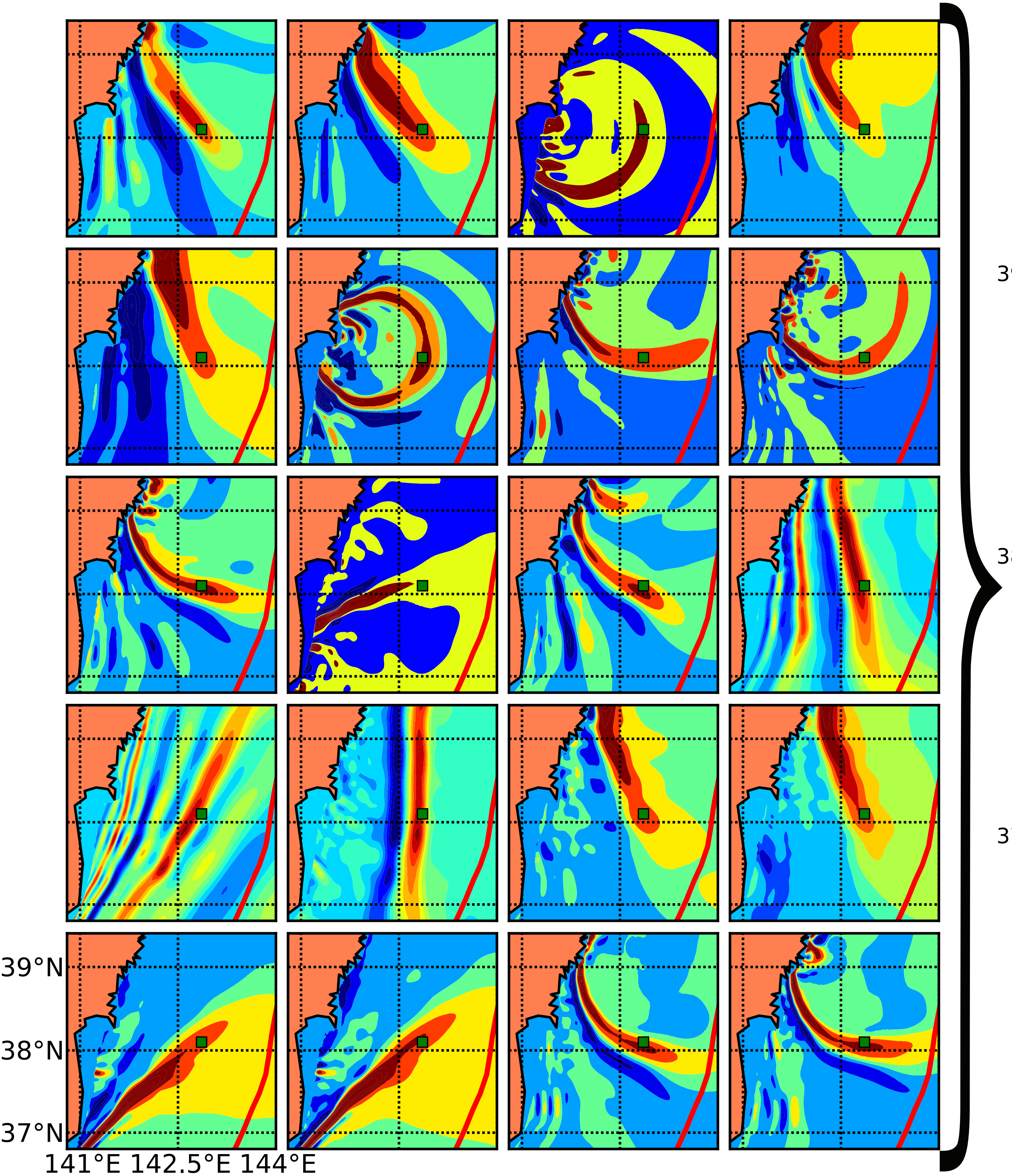
Tsunami TRI: Simulation with encouraging results



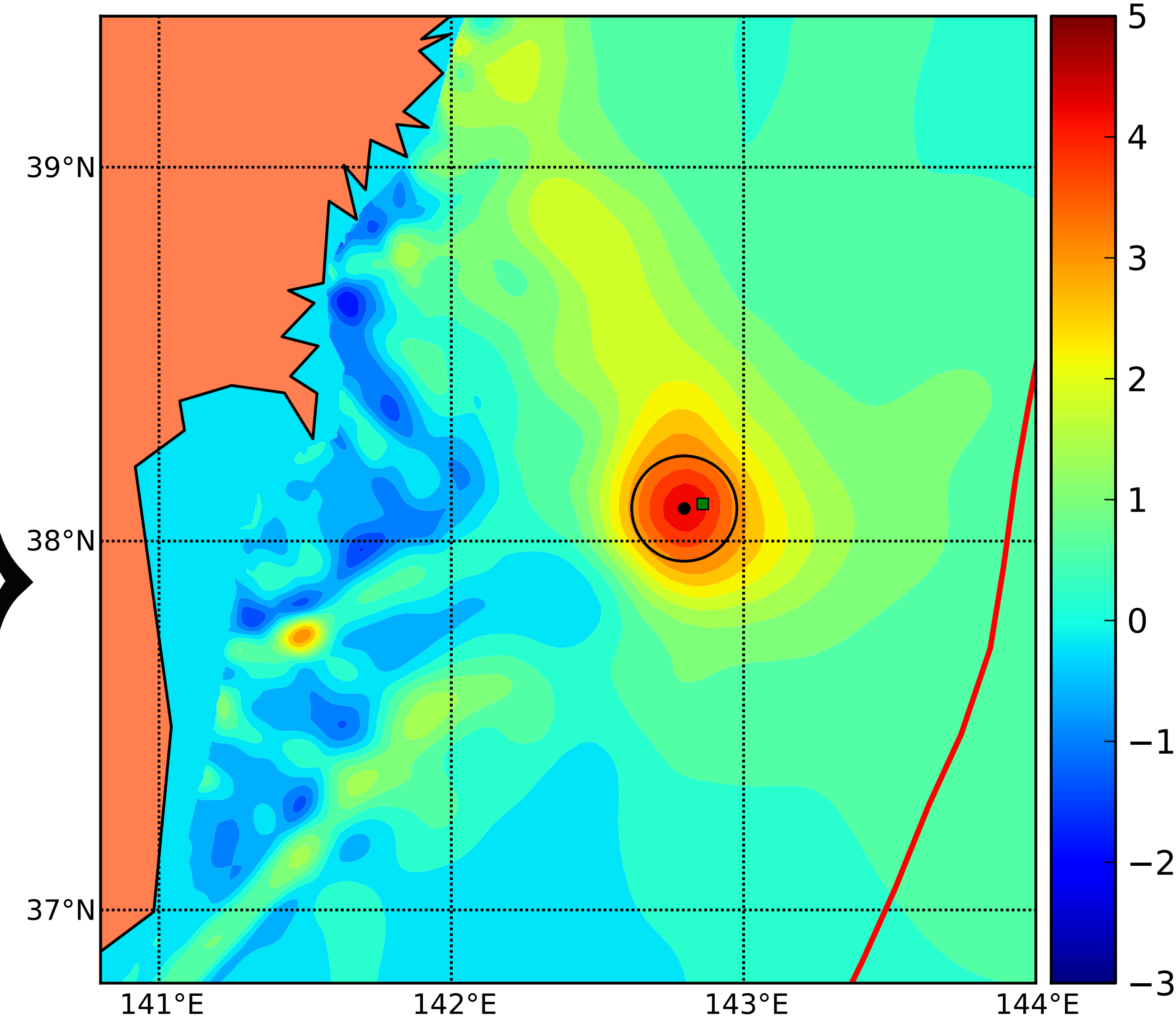


Simple, simulated illustration works…
Problem: Real tsunamis much more complicated

TRI works with uniform scaling… but crude source estimate



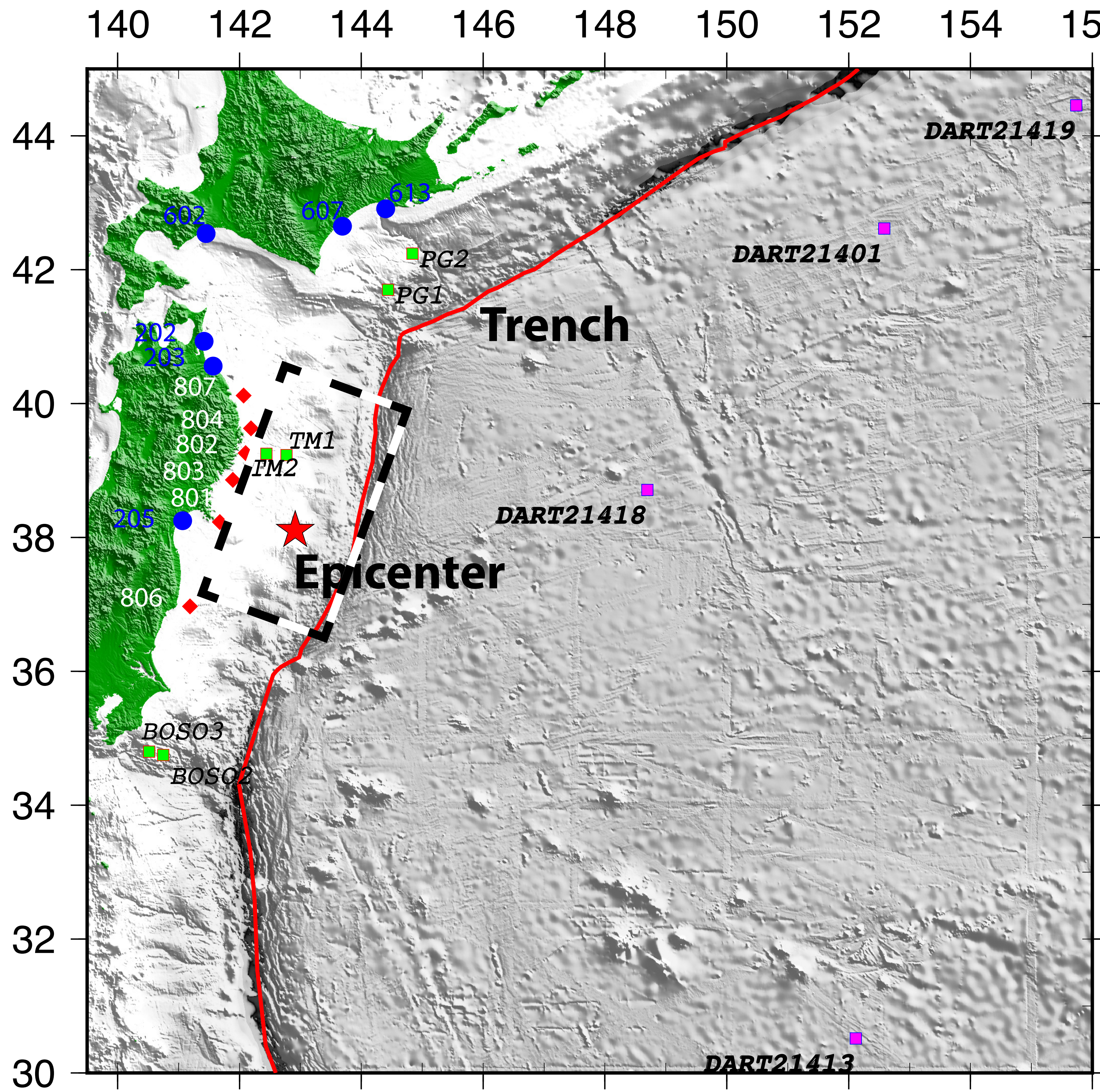
$$\frac{1}{p}\sum_{j=1}^{p}\alpha_{ij}TR_j(\mathbf{s}_i, T-t)$$



Problem: Real tsunamis much more complicated…

2011 Japan tsunami: Ideal to study source process

- Very large tsunami due to giant Mw 9 earthquake
- Unprecedented array of tsunami observations available
- Many published source models for comparison

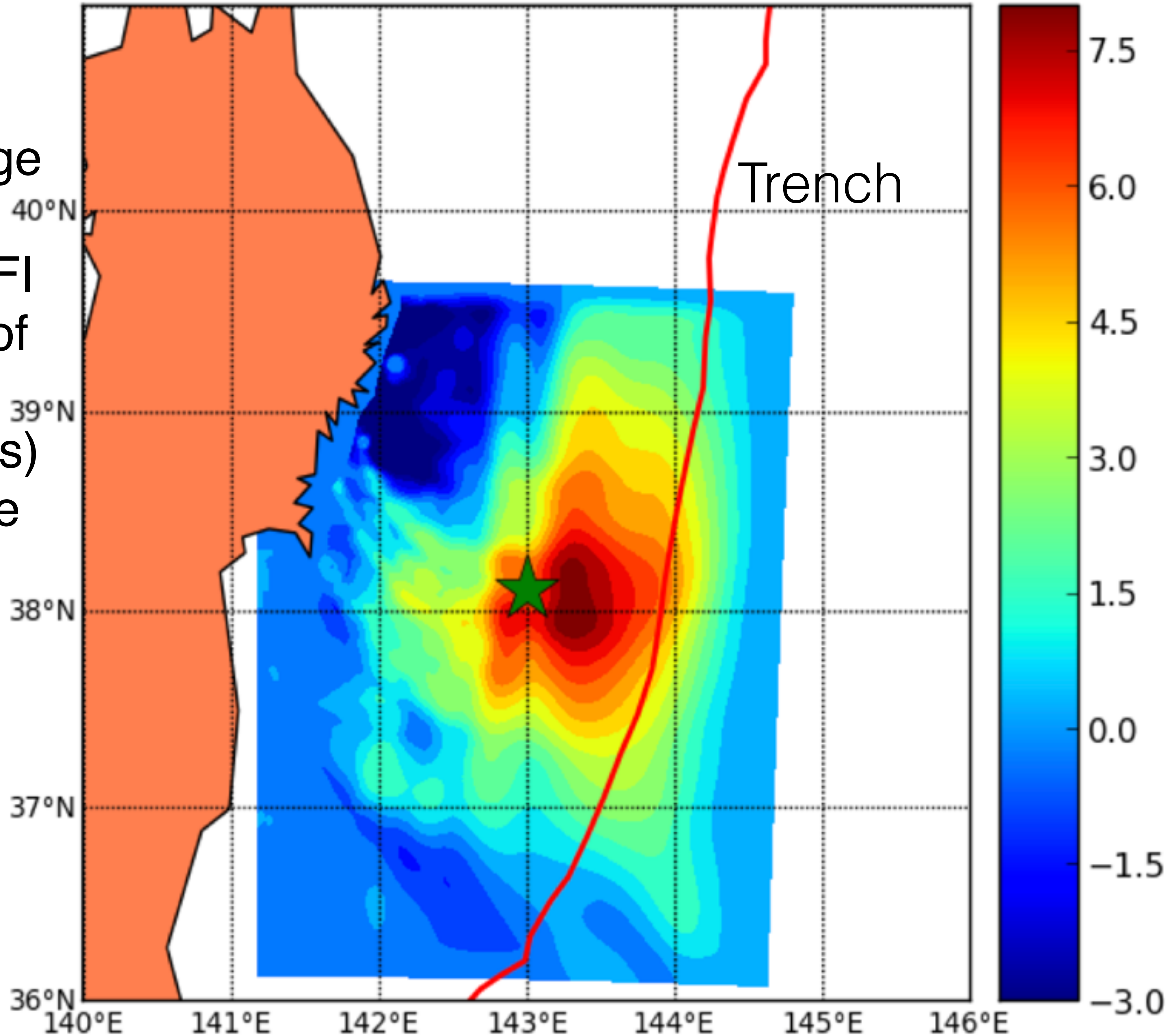


jan.dettmer@anu.edu.au

Basic TRI applied to Japan tsunami: Crude estimate

Problems:

- TRI source image lacks detail evident in the FFI result (sparsity of data causes incomplete focus)
- Artefacts outside source (incomplete destructive interference)



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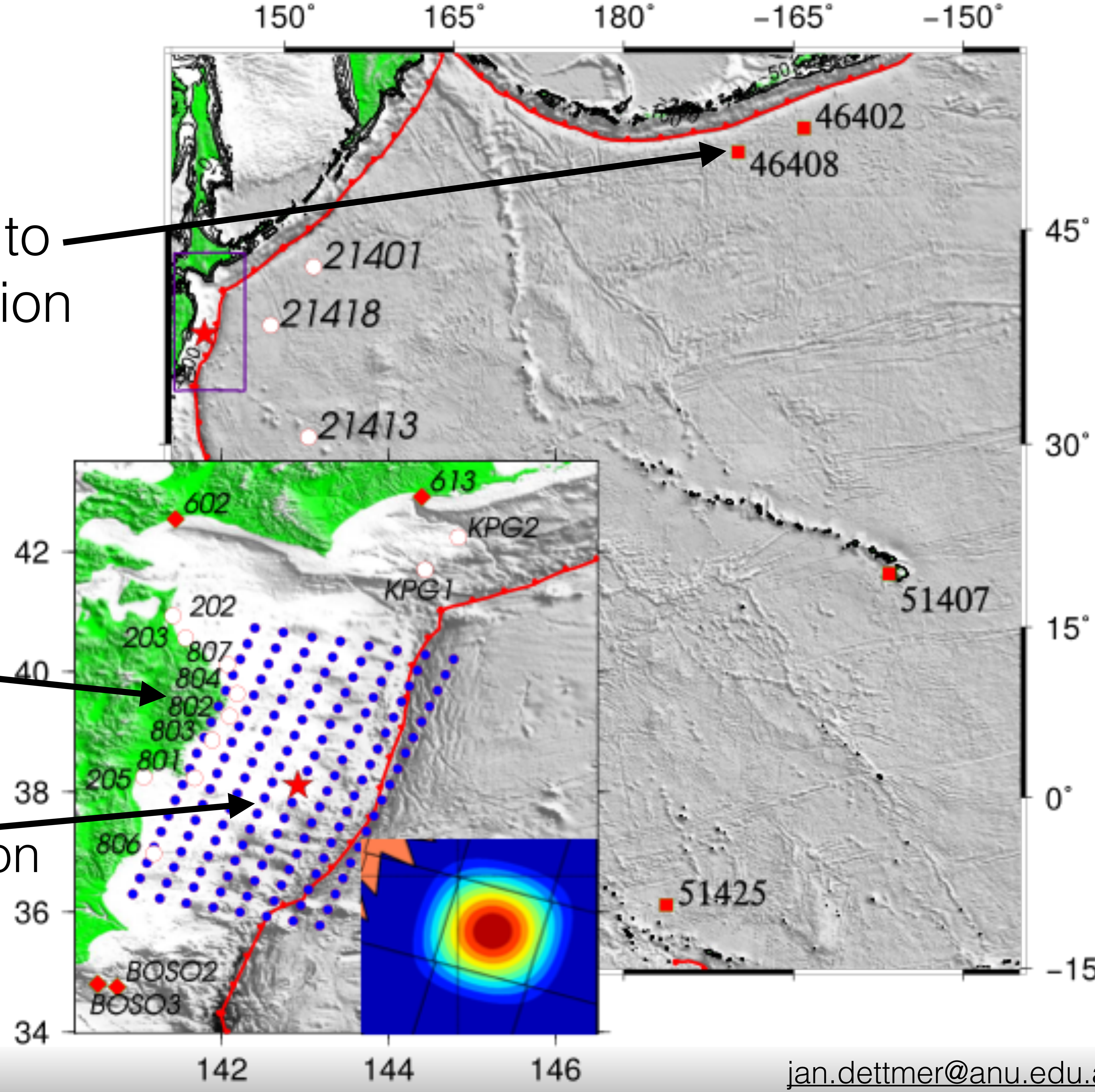
2011 Japan tsunami: Ideal to study source process

Observations to assess prediction

Observations for TRI

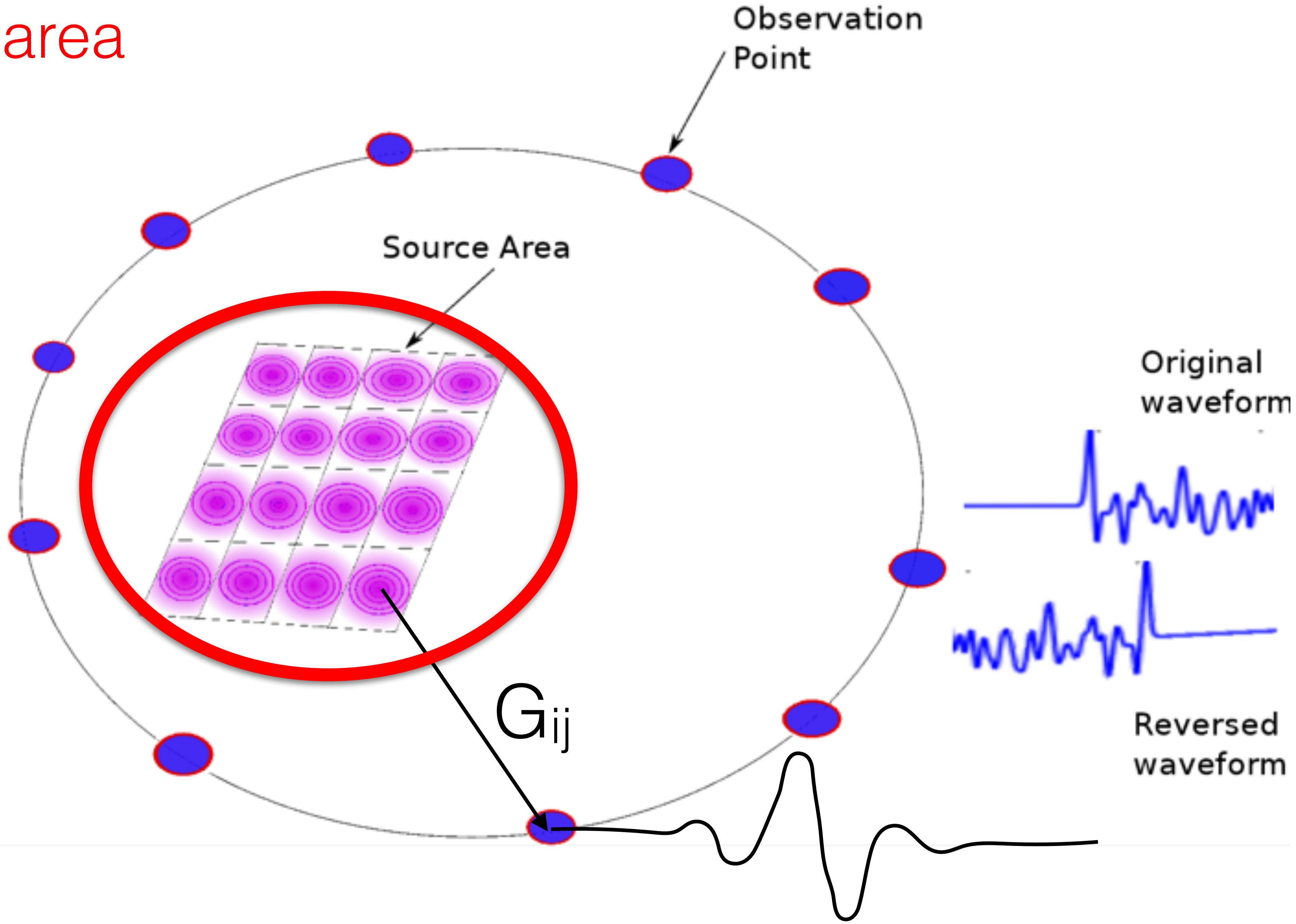
Source discretization



jan.dettmer@anu.edu.au

TRI with Green's function scaling

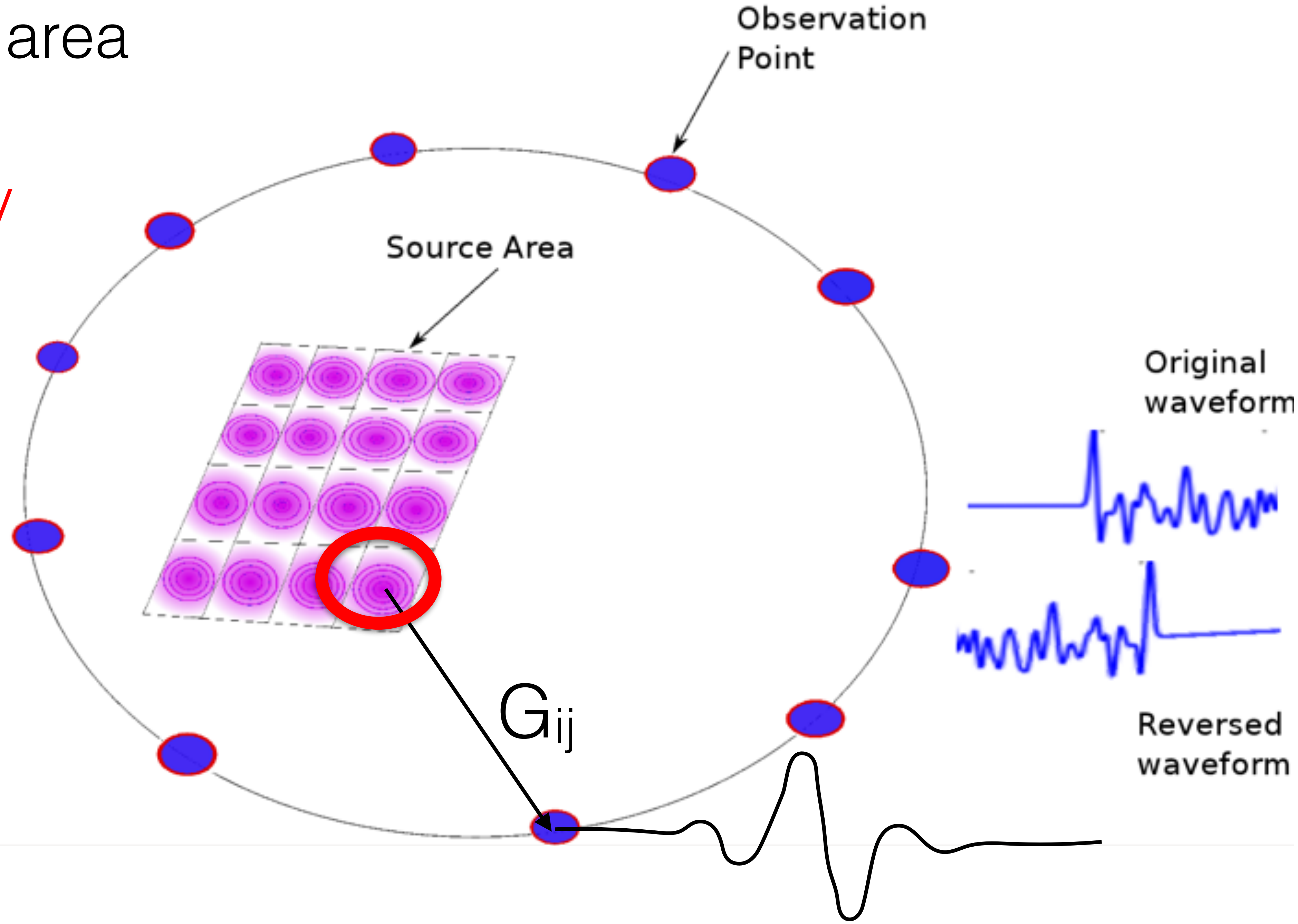
- Divide the source area into subregions



jan.dettmer@anu.edu.au

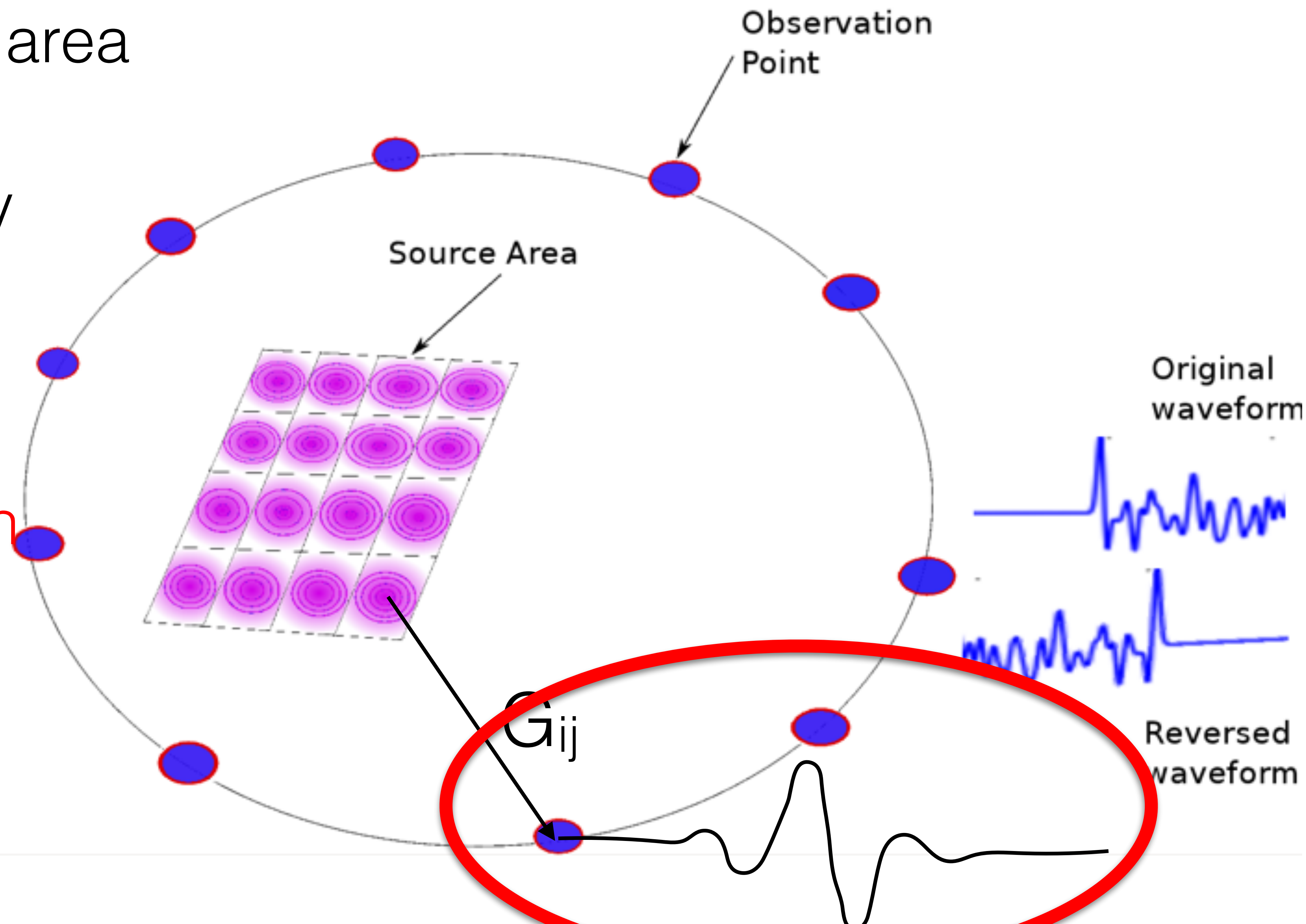
TRI with Green's function scaling

- Divide the source area into subregions.
- Create elementary source over each subregion



TRI with Green's function scaling

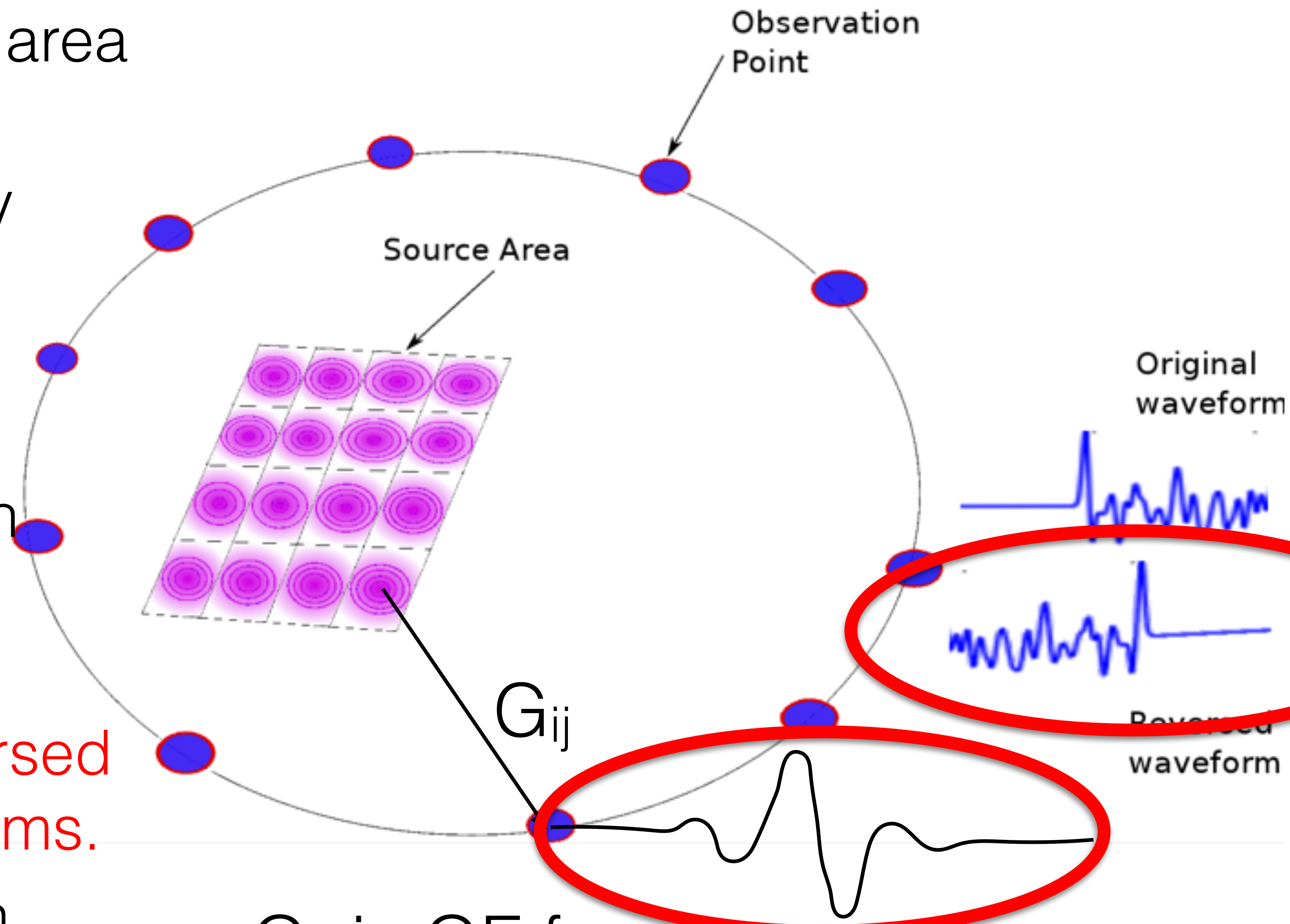
- Divide the source area into subregions.
- Create elementary source over each subregion
- Compute Green's function from each subregion



G_{ij} is GF from source i to receiver j (describe tsunami propagation from i to j; depend on seabed topography)

TRI with Green's function scaling

- Divide the source area into subregions.
- Create elementary source over each subregion
- Compute Green's function from each subregion
- Convolve Green's function with reversed observed waveforms.
- Amplitude at each subregion: Scaled wave height at the final time



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jan.dettmer@anu.edu.au

Green functions: Provide more detail & scaling

Kawakatsu & Montanger (2008): Allow source inversion problem guide choice of TRI scaling factor

$$\frac{\partial J}{\partial \hat{a}_i} = 0$$

$$\left|\sum_{i=1}^{n} G_i(\mathbf{x}_j,\omega)a_i(\omega) - d_j(\omega)\right|^2 \longrightarrow \sum_{i=1}^{n} G^{\star}_{i'}(\mathbf{x}_j,\omega)G_i(\mathbf{x}_j,\omega)\hat{a}_{ij}(\omega) = G^{\star}_{i'}(\mathbf{x}_j,\omega)d_j(\omega)$$

Approximate normal equation matrix as diagonal, with entries:

$$\int G_{i'}(\mathbf{x}_j,\tau)G_i(\mathbf{x}_j,t+\tau)d\tau \approx |G_{ij}|^2\delta_{ii'}$$

Yields an expression for TRI with scaling factor determined:

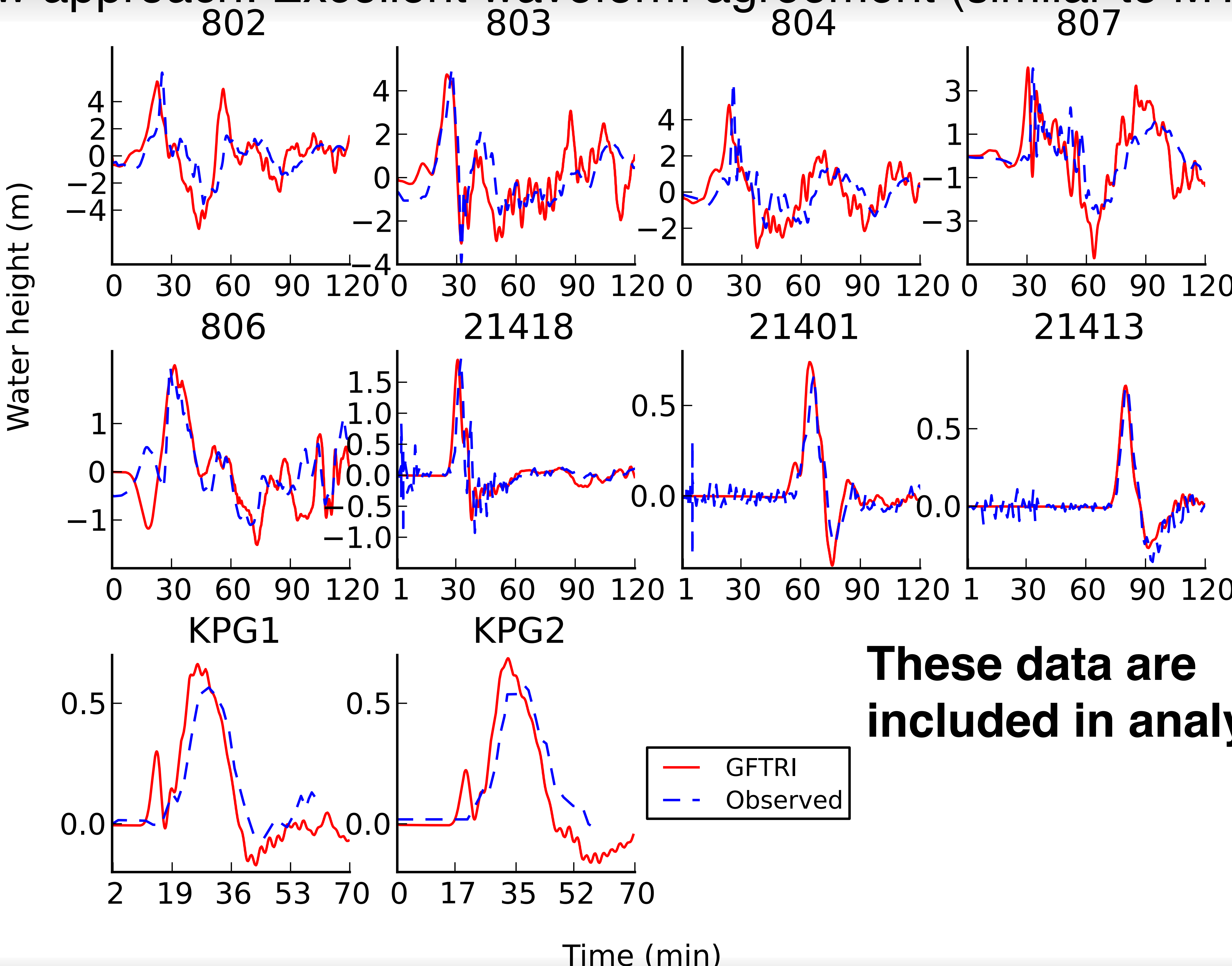
$$\hat{a}_{ij}(\omega) = \frac{1}{|\ G_{ij}\ |^2} G^{\star}_i(\mathbf{x}_j,\omega)d_j(\omega)$$

$$= \frac{e^{i\omega T}}{|\ G_{ij}\ |^2} TR^{\star}_j(\mathbf{s}_i,\omega)$$

I.e., scaling factor is inverse of zero-lag autocorr. of GF

jan.dettmer@anu.edu.au

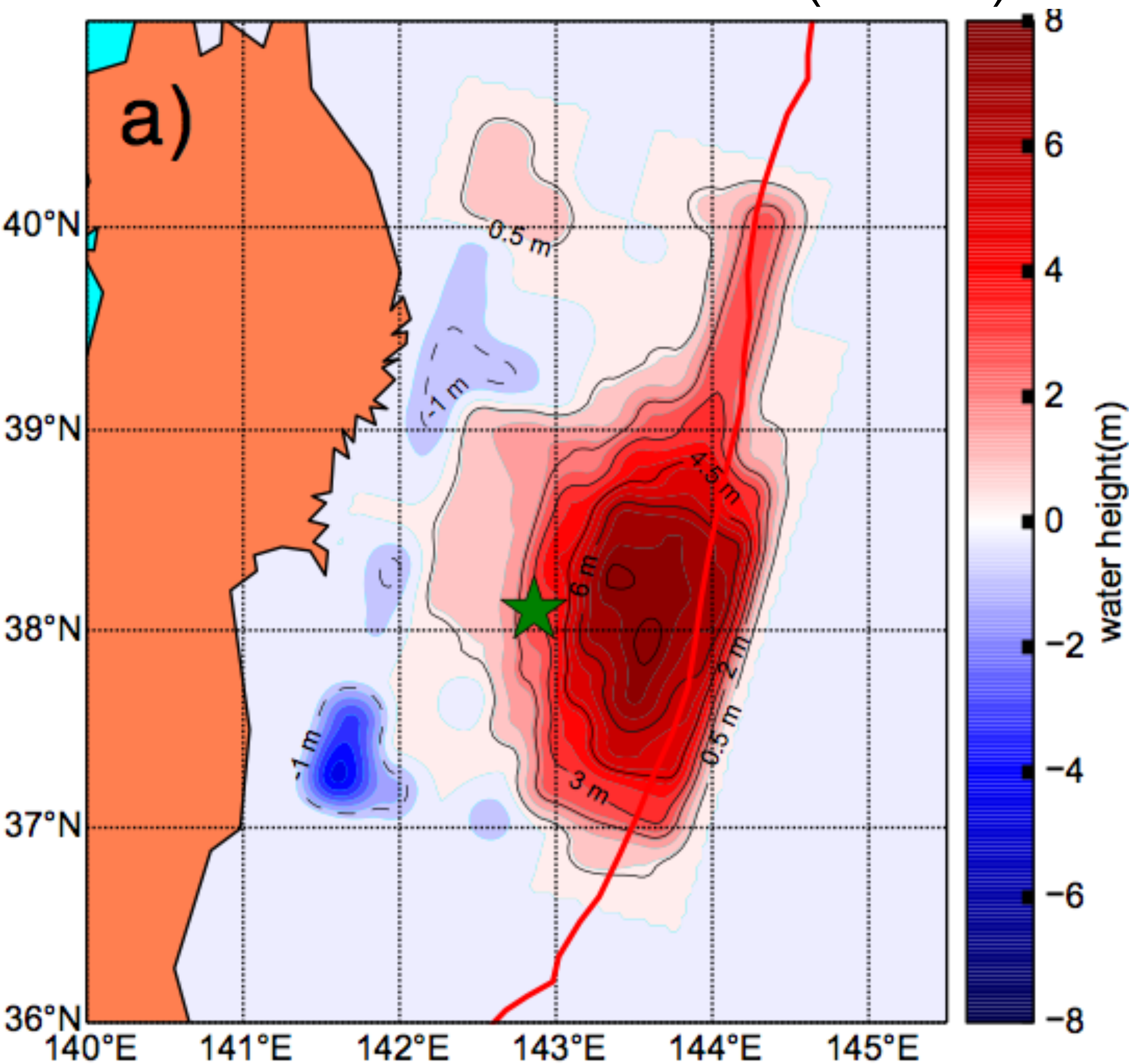
New approach: Excellent waveform agreement (similar to MTW)



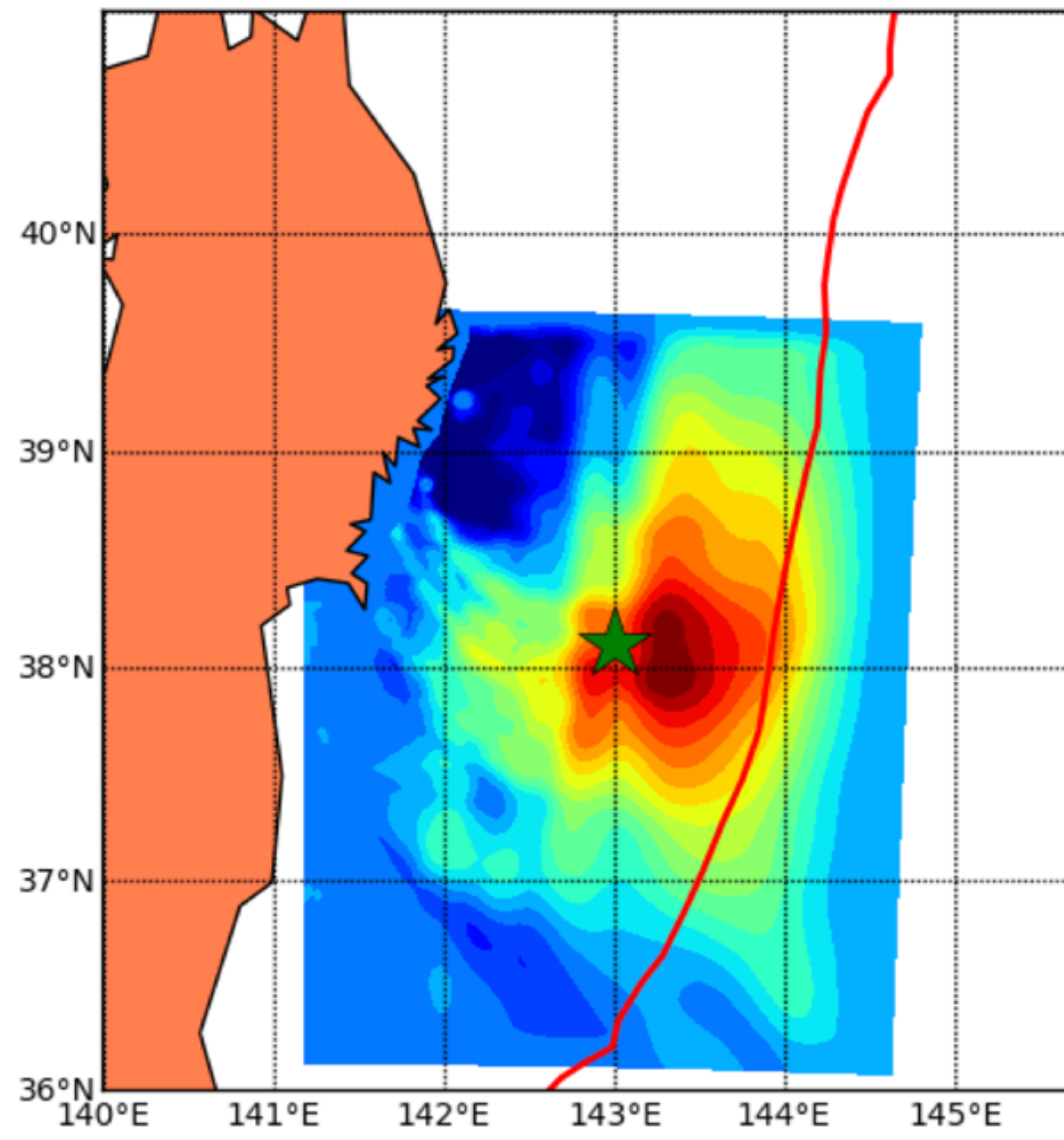
These data are included in analysis

New approach: Similar resolution; Fraction of computational cost, fewer subjective choices

GF-TRI Hossen et al. (2015)



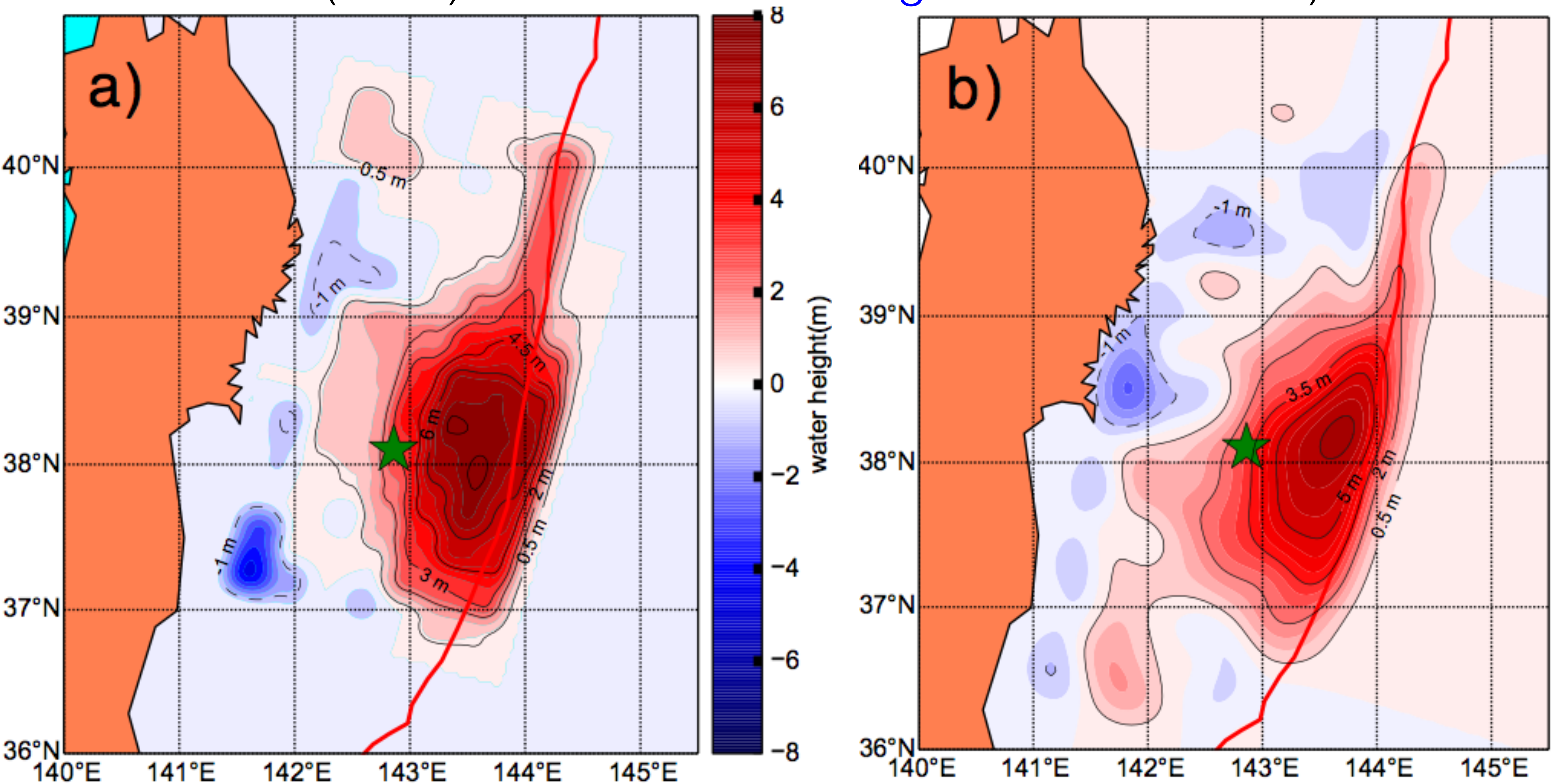
Previous TRI



New approach: Similar resolution; Fraction of computational cost, fewer subjective choices

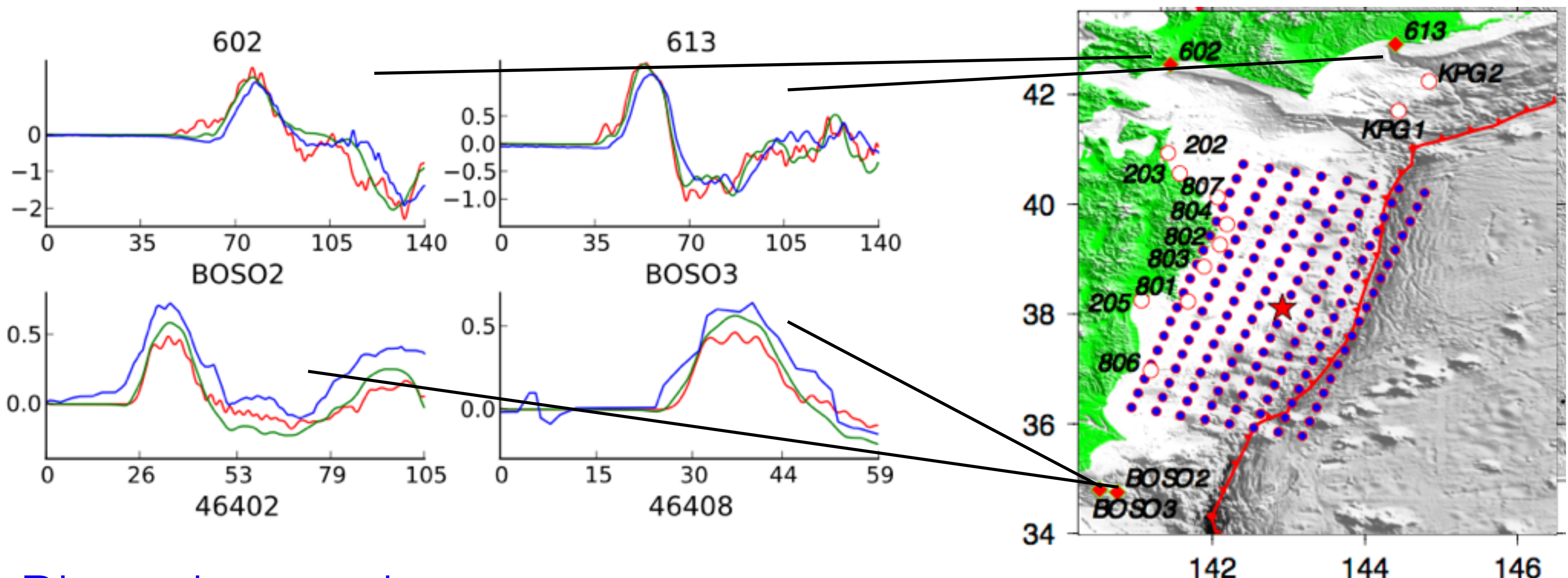
TRI method with GF, Hossen et al. (2015)

Conventional inversion method (multiple time window regularized inversion)



jan.dettmer@anu.edu.au

Predictive power near field:
Similar to conventional but much faster & more objective



Blue: observed

Red: TRI

Green: Conventional

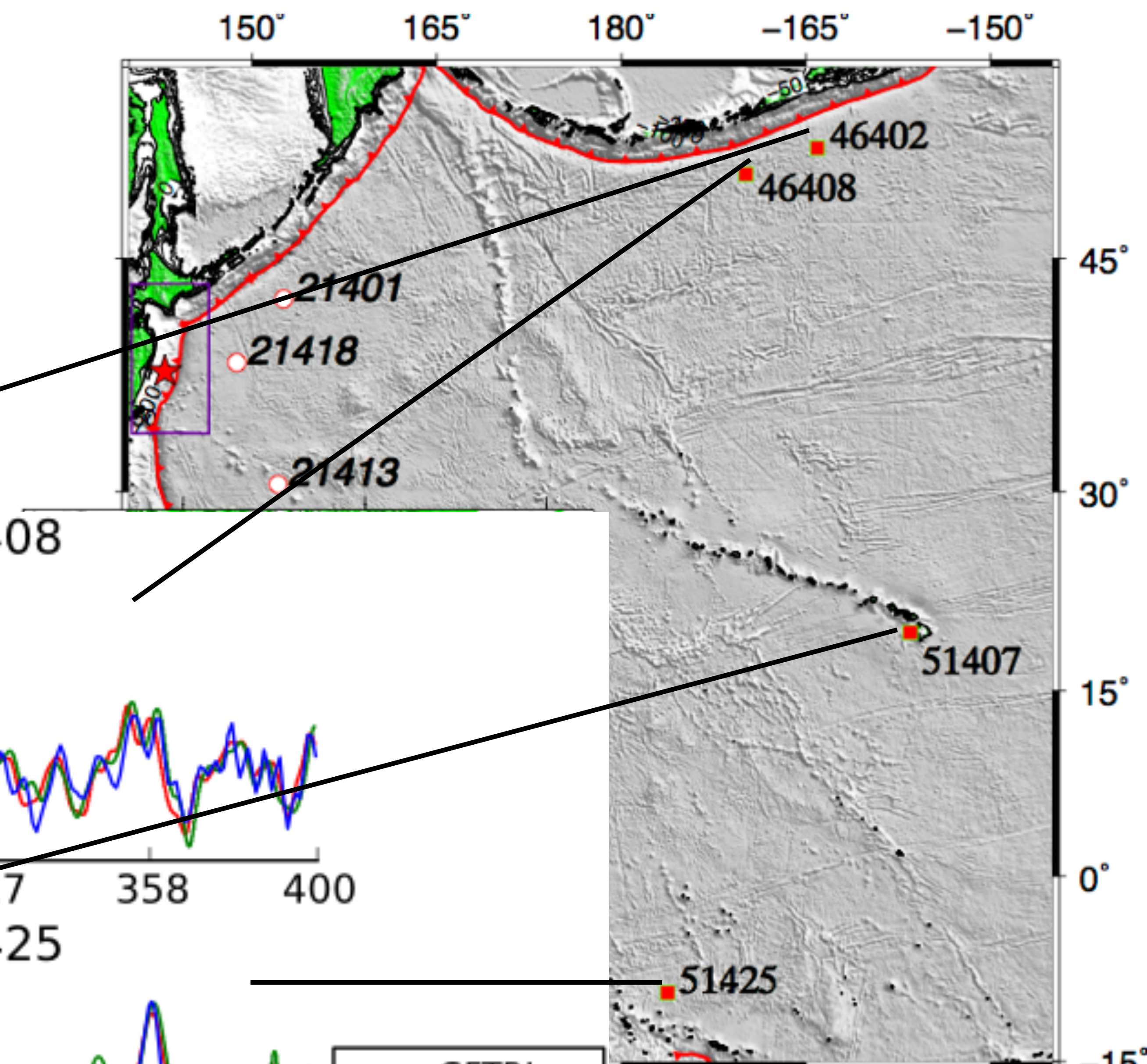
 jan.dettmer@anu.edu.au

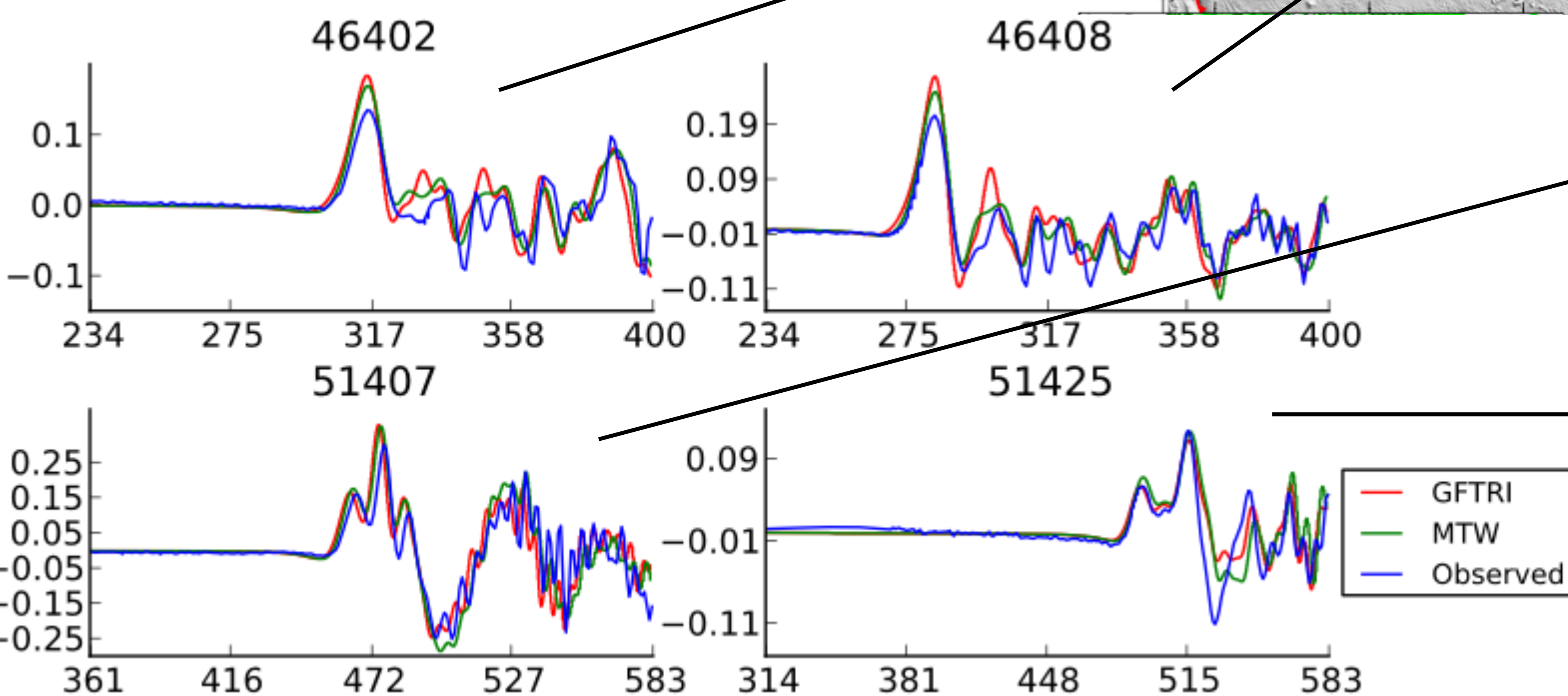
Predictive power far field: Excellent

Blue: observed

Red: TRI

Green: Conventional





Jan Dettmer — ANU
jan.dettmer@anu.edu.au

Importance/future impact: Tsunamis & harbour resonance

- Coastal populations already warned of far-field tsunamis & can escape
- Effects on ports persist for many hours & depend on subtle waveform features that may excite harbour resonance
- Example: Resonance excited in the harbour at Geraldton, Western Australia, by the 2004 Indian Ocean Tsunami

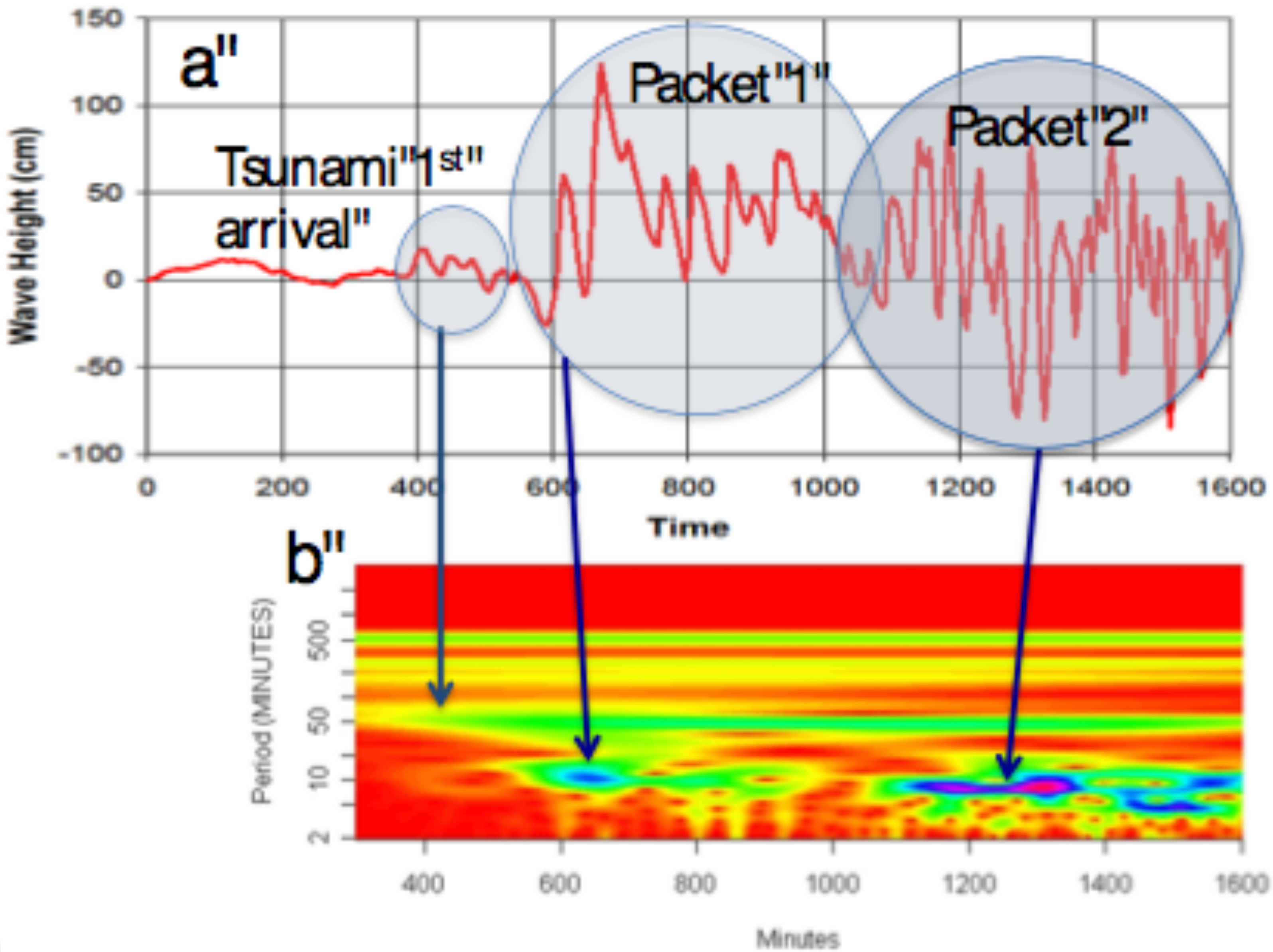


Figure: Geoscience Australia

Summary

- New sea level sensors can be used for time reverse imaging of large tsunami sources
- Tsunami TRI is simple: Average observed waveforms convolved with Green's functions pre-calculated for an array of source points (i.e. no real-time tsunami simulation necessary to estimate source)
- Prediction of far-field waveforms using TRI-determined source is excellent
- Current warning systems work well for alerting distant coastal communities, ports are more difficult to protect:

  Resonances excited by late-arriving phases require better tsunami source estimates which may be forecast effectively using TRI

References:

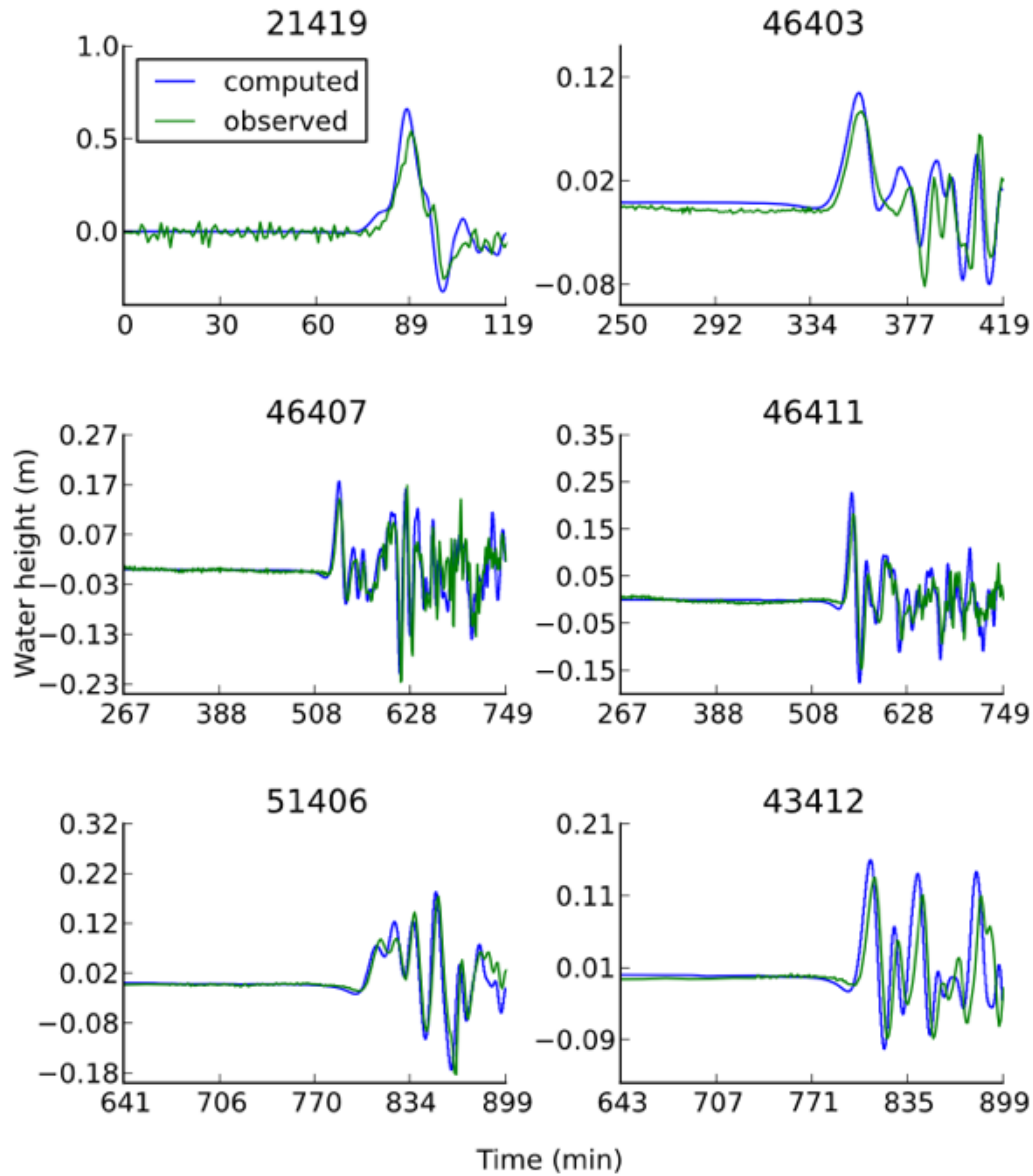
- Hossen et al., (2015). Geophysical Research Letters, 42. doi:10.1002/2015GL065868
- Hossen et al., (2015). Pure and Applied Geophysics, 172, 969-984. doi:10.1007/s00024-014-1014-5

Both available upon request.

jan.dettmer@anu.edu.au

Appendix

More far-field agreements



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